# Model Answer: Summer 2018

**Subject: Mechanics of Structure** | **Sub. Code: 17311**

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**Important Instructions to examiners:**

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more importance. (Not applicable for subject English and Communication Skills.)
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by the candidate and those in the model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and the model answer.
6) In case of some questions credit may be given by judgment on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

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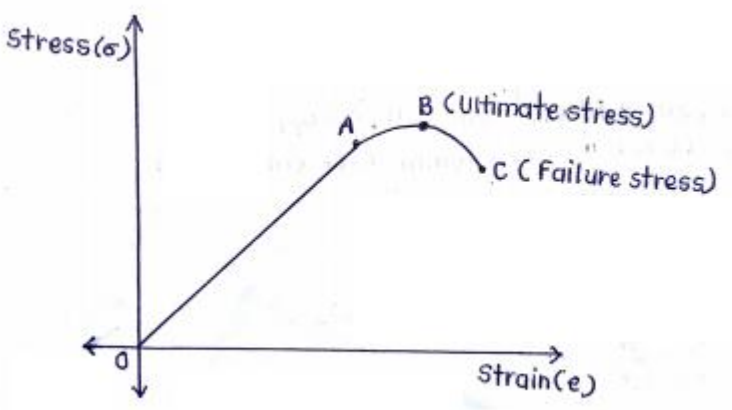
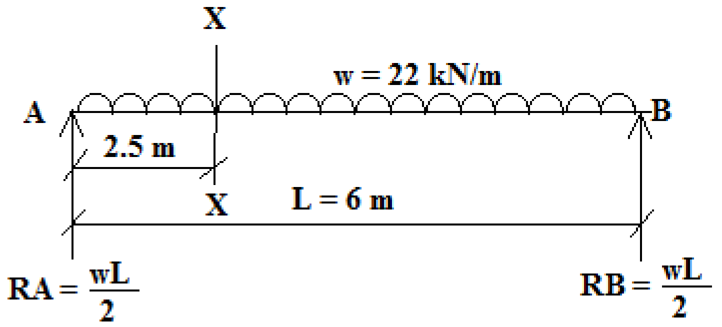
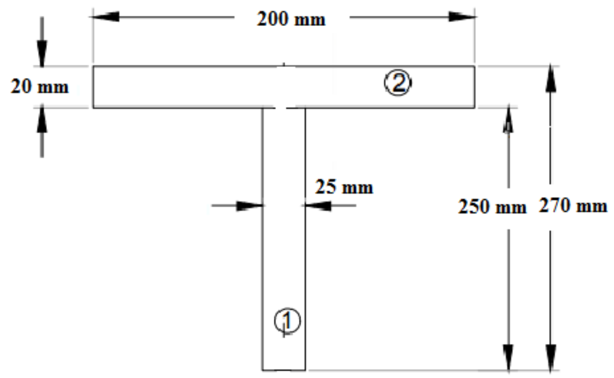
| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 1 | (A) | **Attempt any SIX of the following:** | | 12 |
| | (a) | **State parallel axis theorem.** | | |
| | Ans. | It states that the M. I. of a plane section about any axis parallel to the centroidal axis is equal to the M. I. of the section about the centroidal axis plus the product of the area of the section and the square of the distance between the two axes. | 2 | 2 |
| | (b) | **Define polar moment of inertia.** | | |
| | Ans. | If $I_{XX}$ and $I_{YY}$ are the moment of inertia of a plane section about the two mutually perpendicular axes, then the moment of inertia $I_{ZZ}$ about the third axis ZZ perpendicular to the plane and passing through the intersection of X-X and Y-Y is called as polar moment of inertia. | 2 | 2 |
| | (c) | **Draw stress-strain curve for brittle material showing ultimate stress and failure stress.** | | |
| | Ans. |   Stress - Strain Curve for Brittle Material | 2 | 2 |

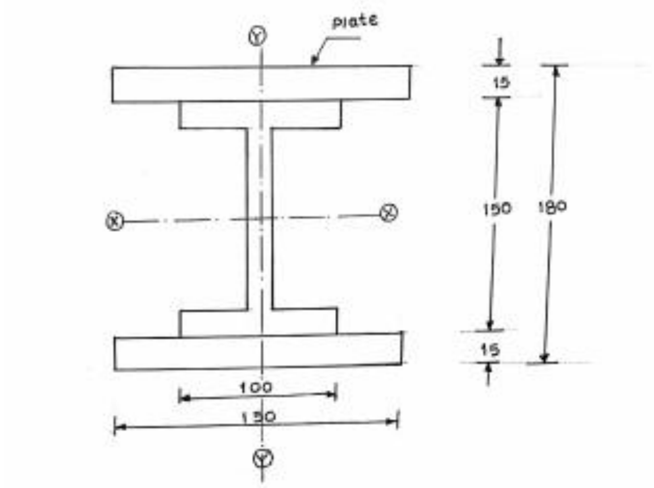
| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 1 | (d) And. | **Define modulus of rigidity. State its unit.**<br>**Modulus of rigidity** : -<br>The ratio of shear stress to shear strain is called as Modulus of Rigidity.<br>**Unit: -** $N/m^2$ Or Pascal Or $N/mm^2$. | 1<br><br>1 | 2 |
| | (e) Ans. | **State any four assumptions made in the theory of long column.**<br>**Assumptions made in the theory of long column are as follows :**<br>1) The material of the column is perfectly homogeneous and isotropic in nature.<br>2) The column is long enough and it fails due to buckling only.<br>3) The load on column is exactly vertical and axial type.<br>4) The self weight of column is neglected.<br>5) The column is initially straight and of uniform lateral dimensions.<br>6) The column is stressed upto the limit of proportionality. | ½ each (any four) | 2 |
| | (f)<br><br>Ans. | **State end conditions for column along with effective lengths (any two).**<br>i. When both end of column are hinged, $L_e = L$<br>ii. When both end of column are fixed, $L_e = \frac{L}{2}$<br>iii. When one end is fixed and other end is hinged, $L_e = \frac{L}{\sqrt{2}}$<br>iv. When one end is fixed and other end is free, $L_e = 2.L$ | 1 each (any two) | 2 |
| | (g) Ans. | **Define strain energy and state its unit.**<br>**Strain Energy:** The energy stored in the material, when it is loaded within its elastic limit, is called as 'Strain Energy'.<br>Unit: N.m or Joule or N.mm. | 1<br><br>1 | 2 |
| | (h) Ans. | **Define proof resilience. Give its expression.**<br>**Proof Resilience:** The maximum energy stored in the material at the point of elastic limit, is called as 'Proof Resilience'.<br>Expression:<br>$U_{max} = \frac{\sigma^2_{max} \times V}{2 \times E}$ | 1<br><br>1 | 2 |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 1 | (B) | **Attempt any TWO of the following:** | | 8 |
| | (a) | **A simply supported beam of span 10 m carries a udl of 100 kN/m over its entire span. Calculate the modulus of section required, if its bending stress is not to exceed 190 MPa.** | | |
| | Ans. | Given:<br>$L = 10\text{ m} = 10 \times 10^3\text{ mm}$<br>Simply supported<br>$w = 100\text{ kN/m} = 100\text{ N/mm}$ over entire span<br>$\sigma b_{max} = 190\text{ MPa} = 190\text{ N/mm}^2$<br>To find:<br>Z = ? | | |
| | | Solution:<br>We know, the flexural formula,<br>$\frac{M}{I} = \frac{\sigma_b}{y}$<br>$\sigma_b = \frac{M}{I} \times y = \frac{M}{\left(\frac{I}{y}\right)} = \frac{M}{Z}$ | | 4 |
| | | $Z = \frac{M}{\sigma_b}$ | 1 | |
| | | Here, $M = \frac{w \times L^2}{8}$ | 1 | |
| | | $= \frac{100 \times 10000^2}{8} = 125 \times 10^7\text{ N.mm}$ | 1 | |
| | | $Z = \frac{125 \times 10^7}{190}$<br>$Z = 6578947.36\text{ mm}^3$ | 1 | |
| | | $\boxed{Z = 6.578 \times 10^6\text{ mm}^3}$ | | |
| | (b) | **A simply supported beam carries a udl of 22 kN/m over the entire span of 6 m. Beam has circular c/s of 150 mm diameter. Determine the maximum shear stress at a section 2.5 m from the support.** | | |
| | Ans. | Given:<br>w = 22 kN/m over entire span<br>L = 6 m<br>Simply supported<br>d = 150 mm<br>To find:<br>$q_{max}$ at 2.5 m from support = ? | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **Q. 1** | | Solution: <br>   <br> Shear force at a section x-x 2.5 m from support <br> $S = \frac{w \times L}{2} - (w \times 2.5)$ <br> $= \frac{22 \times 6}{2} - (22 \times 2.5)$ <br> $S = 11\text{kN}$ <br> $S = 11 \times 10^3 \text{N}$ | **1** | **4** |
| | | $q_{avg} = \frac{S}{A}$ <br> $= \frac{11 \times 10^3}{\frac{\pi}{4} \times 150^2}$ <br> $= 0.622\ \text{N/mm}^2$ | **1** | |
| | | $q_{max} = \frac{4}{3} \times q_{avg}$ | **1** | |
| | | $= \frac{4}{3} \times 0.622$ <br> $\boxed{q_{max} = 0.83\ \text{N/mm}^2}$ | **1** | |
| | | **OR** | | |
| | | $A = \frac{\pi}{8} \times d^2 = \frac{\pi}{8} \times 150^2 = 8835.729\ \text{mm}^2$ <br> $\bar{y} = \frac{4 \times R}{3 \times \pi} = \frac{4 \times 75}{3 \times \pi} = 31.831\ \text{mm}$ <br> $b = d = 150\ \text{mm}$ <br> $I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} \times 150^4 = 24850488.76\ \text{mm}^4$ <br> $q_{\text{ at 2.5m}} = \frac{S \times A \times \bar{y}}{b \times I} = \frac{11 \times 10^3 \times 8835.729 \times 31.831}{150 \times 24850488.76} = 0.83\ \text{N/mm}^2$ | **2** | |
| | | $\boxed{q_{\text{ at 2.5m}} = 0.83\ \text{N/mm}^2}$ | **1** | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 1 | (c) | **A column of square section 150 mm x 150 mm is 3 m long. If it is fixed at its both ends, find the Euler's critical load on the column. Take E = 175 GPa.** | | |
| | Ans. | Given: Square column, both ends fixed, L = 3 m = 3000 mm, $E = 175\ GPa = 175 \times 10^3\ N/mm^2$. To find: $P_e = ?$ | | |
| | | Solution: | | |
| | | Effective length $(L_e) = \frac{L}{2} = \frac{3000}{2} = 1500\ mm$ | 1 | |
| | | Moment of Inertia $(I_{xx}) = (I_{yy}) = (I_{min})$ | | 4 |
| | | $= \frac{b^4}{12} = \frac{150^4}{12} = 42.187 \times 10^6\ mm^4$ | 1 | |
| | | Euler's critical load $(P_E) = \frac{\pi^2 \times E \times I_{min}}{(L_e)^2}$ | 1 | |
| | | $= \frac{\pi^2 \times 175 \times 10^3 \times 42.187 \times 10^6}{(1500)^2}$ | | |
| | | $\boxed{P_E = 32.384 \times 10^6\ N}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 2 | | **Attempt any TWO of the following:** | | 16 |
| | (a) | **Calculate the moment of inertia about its both centroidal axes of a T – section having flange 200 mm x 20 mm and web 250 mm x 25 mm. Overall depth is 270 mm.** | | |
| | Ans. | Given:   To find: $I_{xx}$ and $I_{yy}$ = ? | | |
| | | Solution:<br>As given T-section is symmetrical about Y-Y axis,<br>$\bar{x}=\frac{\text{Flange width}}{2}=\frac{200}{2}=100\text{ mm}$<br>$A_1=250\times25=6250\text{ mm}^2$<br>$A_2=200\times20=4000\text{ mm}^2$<br>$y_1=\frac{250}{2}=125\text{ mm}$<br>$y_2=250+\frac{20}{2}=260\text{ mm}$<br>$\bar{y}=\frac{(A_1\times y_1)+(A_2\times y_2)}{(A_1+A_2)}$<br>$=\frac{(6250\times125)+(4000\times260)}{(6250+4000)}$ | | 8 |
| | | $=177.683\text{ mm from bottom}$ | 1 | |
| | | $I_{xx}=I_{xx_1}+I_{xx_2}$<br>$=(I_{G_1}+A_1\times h_1^2)+(I_{G_2}+A_2\times h_2^2)$ | 1 | |
| | | Here, $h_1=\bar{y}-y_1=177.68-125=52.68\text{ mm}$<br>$h_2=y_2-\bar{y}=260-177.68=82.32\text{ mm}$ | 1 | |
| | | $I_{xx}=\left(\frac{bd^3}{12}+A_1\times h_1^2\right)+\left(\frac{bd^3}{12}+A_2\times h_2^2\right)$<br>$=\left(\frac{25\times250^3}{12}+(6250\times52.68^2)\right)+\left(\frac{200\times20^3}{12}+(4000\times82.32^2)\right)$ | 1 | |
| | | $\boxed{I_{xx}=77.136\times10^6\text{ mm}^4}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 2 | | $I_{yy} = I_{yy_1} + I_{yy_2}$ | 1 | |
| | | $= \left[\frac{db^3}{12}\right]_1 + \left[\frac{db^3}{12}\right]_2$ | 1 | |
| | | $= \frac{250\times25^3}{12} + \frac{20\times200^3}{12}$ | | |
| | | $\boxed{I_{yy} = 13.658\times10^6 \text{ mm}^4}$ | 1 | |
| | (b) | **A steel stanchion is built up of 100 mm x 150 mm RSJ with one 150 mm x 15 mm plate riveted to each flange. The overall depth of the stanchion is 180 mm. Calculate M.I. about its both centroidal axes. Properties of RSJ are A = 2167 mm$^2$, $I_{xx}$ = 8.40 x 10$^6$ mm$^4$, $I_{yy}$ = 0.95 x 10$^6$ mm$^4$.** | | |
| | Ans. | Given:<br>A = 2167 mm$^2$<br>$I_{xx}$ = 8.40 x 10$^6$mm$^4$<br>$I_{yy}$ = 0.95 x 10$^6$ mm$^4$<br>To find:<br>$I_{xx}$ and $I_{yy}$ of builtup section ? | | |
| | | Solution:<br> | | |
| | | $I_{xx} = I_{xx} \text{ of RSJ} + 2(I_{xx} \text{ of plate})$ | 1 | 8 |
| | | $= I_{xx} \text{ of RSJ} + 2\left(\frac{b\times d^3}{12} + (A\times h^2)\right)$ | 1 | |
| | | $= (8.40\times10^6) + 2\left(\frac{150\times15^3}{12} + \left(2250\times\left(75+\frac{15}{2}\right)^2\right)\right)$ | 2 | |
| | | $\boxed{I_{xx} = 39.113\times10^6 \text{ mm}^4}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
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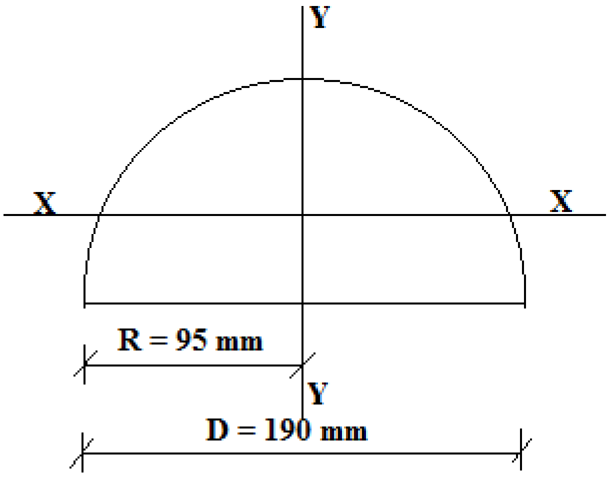
| **Q. 2** | | $I_{yy} = I_{yy} \text{ of RSJ} + 2(I_{yy} \text{ of plate})$ | **1** | |
| | | $= I_{yy} \text{ of RSJ} + 2\left(\frac{d \times b^3}{12}\right)$ | **1** | |
| | | $= (0.95 \times 10^6) + 2\left(\frac{15 \times 150^3}{12}\right)$ | | |
| | | $\boxed{I_{yy} = 9.387 \times 10^6 \text{ mm}^4}$ | **1** | |
| | **(c)-(i)** | **Calculate the M.I. of a semi circle of diameter 190 mm about its both centroidal axes.** | | |
| | **Ans.** | Given: Semicircle of dia. (d) = 190 mm<br>To find: $I_{xx}$ and $I_{yy}$ = ? | | |
| | | Solution:<br><br> | | |
| | | $I_{xx} = 0.11 \times R^4 = 0.11 \times 95^4$ | **1** | |
| | | $\boxed{I_{xx} = 8.959 \times 10^6 \text{mm}^4}$ | **1** | **4** |
| | | $I_{yy} = \frac{\pi D^4}{128} = \frac{\pi \times 190^4}{128}$ | **1** | |
| | | $\boxed{I_{yy} = 31.985 \times 10^6 \text{mm}^4}$ | **1** | |
| | | OR | | |
| | | $I_{yy} = \frac{\pi R^4}{8} = \frac{\pi \times 95^4}{8}$ | **1** | |
| | | $\boxed{I_{yy} = 31.985 \times 10^6 \text{mm}^4}$ | **1** | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
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| Q. 2 | (c)(ii) | **A force of 50 kN is required to punch a circular hole of 20 mm diameter in a metal plate having 3.5 mm thickness. Calculate shear stress and compressive stress developed in the punching rod.** | | |
| | Ans. | Given: $P = 50\text{ kN} = 50 \times 10^3\text{ N}$, $d = 20\text{ mm}$, $t = 3.5\text{ mm}$<br>To find: $q = ?$, $\sigma_{comp} = ?$ | | |
| | | Solution: | | 4 |
| | | $q = \dfrac{P}{\pi \times d \times t}$ | 1 | |
| | | $= \dfrac{50 \times 10^3}{\pi \times 20 \times 3.5}$ | | |
| | | $\boxed{q = 227.364\text{ N/mm}^2}$ | 1 | |
| | | $\sigma_{comp} = \dfrac{P}{A}$ | 1 | |
| | | $= \dfrac{50 \times 10^3}{\frac{\pi}{4} \times 20^2}$ | | |
| | | $\boxed{\sigma_{comp} = 159.155\text{ N/mm}^2}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| **Q. 3** | | **Attempt any TWO of the following:** | | **16** |
| | **(a)** | **A hollow circular steel tube of external diameter 400 mm and uniform thickness of 25 mm throughout is filled in with concrete from inside. Calculate the total axial compressive load the column can support if permissible stress in concrete and steel are 4 MPa and 130 MPa respectively. The modular ratio is 18.** | | |
| | **Ans.** | Given: D = 400 mm, t = 25 mm, $\sigma_c$ = 4 MPa, $\sigma_S$ = 130 MPa, m = 18. To find: P = ? | | |
| | | Solution: $d = (D - 2\times t) = (400 - 2\times 25) = 350\,mm$ | | |
| | | $A_s = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(400^2 - 350^2) = 29452.43\,mm^2$ | 1 | |
| | | $A_c = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}\times 350^2 = 96211.275\,mm^2$ | 1 | |
| | | But, $\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$; $\sigma_s = \left(\frac{E_s}{E_c}\right)\times\sigma_c = m\times\sigma_c$; $\sigma_s = 18\times 4$; $\sigma_s = 72\,MPa < 130\,MPa$ | 1 | 8 |
| | | If $\sigma_s = 130\,N/mm^2$ is taken, $\sigma_c = \frac{\sigma_s}{18} = \frac{130}{18} = 7.22\,N/mm^2 > 4\,MPa$ | 1 | |
| | | $\therefore$ Take $\sigma_s = 72\,N/mm^2$ | 1 | |
| | | $P = P_s + P_c$ | 1 | |
| | | $P = (\sigma_s\times A_s) + (\sigma_c\times A_c) = (72\times 29452.43) + (4\times 96211.275) = 2505420.06\,N$ | 1 | |
| | | $\boxed{P = 2505.42\,kN}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
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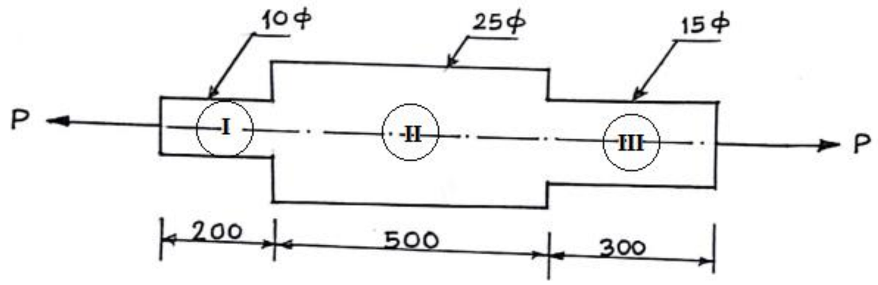
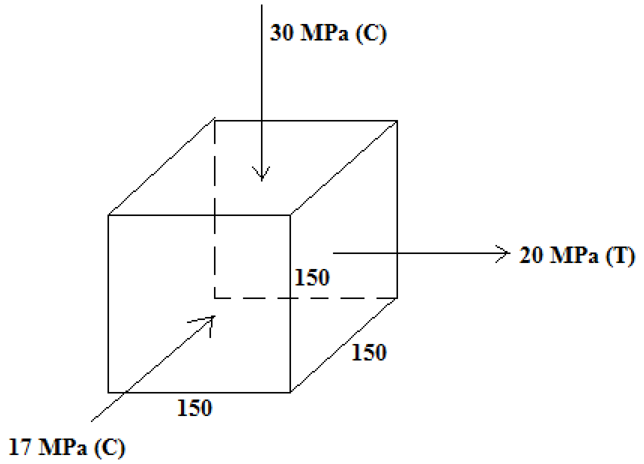
| Q. 3 | (b) | **A circular bar of 1000 mm length has cross – section as given below : First 200 mm has a diameter 10 mm, second 500 mm has a diameter 25 mm and the last 300 mm has diameter of 15 mm. Determine the maximum axial pull which the bar may be subjected if the maximum stress is limited to 150 MPa. Find total elongation of the bar. Take E = 200 GPa.** | | |
| | **Ans.** | Given: $\sigma_{max} = 150$ MPa, $E = 200$ GPa. To find: $P_{max} = ?$, $\delta_L = ?$ | | |
| | | Solution:   | | |
| | | $A_I = \frac{\pi}{4}\times 10^2 = 78.54\ mm^2$ <br> $A_{II} = \frac{\pi}{4}\times 25^2 = 490.87\ mm^2$ <br> $A_{III} = \frac{\pi}{4}\times 15^2 = 176.71 mm^2$ | 1 | |
| | | $P_{max} = \sigma_{max} \times A_I$ | 1 | 8 |
| | | $= 150\times 78.54$ <br> $= 11780.97\ N$ | 1 | |
| | | $\boxed{P_{max} = 11.781 kN}$ | | |
| | | $\delta_L = (\delta_L)_I + (\delta_L)_{II} + (\delta_L)_{III}$ | 1 | |
| | | $= \left(\frac{PL}{AE}\right)_I + \left(\frac{PL}{AE}\right)_{II} + \left(\frac{PL}{AE}\right)_{III}$ | 1 | |
| | | $= \frac{P}{E}\left[\left(\frac{L}{A}\right)_I + \left(\frac{L}{A}\right)_{II} + \left(\frac{L}{A}\right)_{III}\right]$ | 1 | |
| | | $= \frac{11780.97}{200\times 10^3}\times\left[\left(\frac{200}{78.54}\right)+\left(\frac{500}{490.87}\right)+\left(\frac{300}{176.71}\right)\right]$ | 1 | |
| | | $\boxed{\delta_L = 0.31 mm}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 3 | (c) | **A cube of 150 mm side is acted upon by stress along the three directions as 20 MPa (tensile), 30 MPa (compressive) and 17 MPa (compressive). Calculate strains in all the three directions and change in the volume of the cube. Take E = 210 GPa and μ = 0.29.** | | |
| | Ans. | Given: $\sigma_x = +20\ N/mm^2$, $\sigma_y = -30\ N/mm^2$, $\sigma_z = -17\ N/mm^2$; a = 150 mm, E = 210 GPa, μ = 0.29. To find: $e_x, e_y, e_z, \delta_v = ?$ | | |
| | | Solution:   | | |
| | | $e_x = \left(\frac{\sigma_x}{E}\right) - \left(\mu \times \frac{\sigma_y}{E}\right) - \left(\mu \times \frac{\sigma_z}{E}\right)$ | 1 | 8 |
| | | $= \frac{1}{E}\left(\sigma_x - \mu\times\sigma_y - \mu\times\sigma_z\right)$ | | |
| | | $= \frac{1}{210\times10^3}\left(20 + (0.29\times30) + (0.29\times17)\right)$ | | |
| | | $\boxed{e_x = 1.60\times10^{-4}}$ | 1 | |
| | | $e_y = \left(\frac{\sigma_y}{E}\right) - \left(\mu \times \frac{\sigma_z}{E}\right) - \left(\mu \times \frac{\sigma_x}{E}\right)$ | 1 | |
| | | $= \frac{1}{E}\left(\sigma_y - \mu\times\sigma_z - \mu\times\sigma_x\right)$ | | |
| | | $= \frac{1}{210\times10^3}\left(-30 + (0.29\times17) - (0.29\times20)\right)$ | | |
| | | $\boxed{e_y = -1.468\times10^{-4}}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
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| Q. 3 | | $e_z=\left(\frac{\sigma_z}{E}\right)-\left(\mu\times\frac{\sigma_x}{E}\right)-\left(\mu\times\frac{\sigma_y}{E}\right)$ | 1 | |
| | | $=\frac{1}{E}\left(\sigma_z\ -\mu\times\sigma_x\ -\mu\times\sigma_y\right)$ | | |
| | | $=\frac{1}{210\times10^3}\left(-17\ -(0.29\times20)\ +(0.29\times30)\right)$ | | |
| | | $\boxed{e_z=-6.714\times10^{-5}}$ | 1 | |
| | | $\frac{\delta_v}{V}=e_x+e_y+e_z$ | | |
| | | $\delta_v=\left(e_x+e_y+e_z\right)\times V$ | 1 | |
| | | $=\left[\left(1.60\times10^{-4}\right)+\left(-1.468\times10^{-4}\right)+\left(-6.714\times10^{-5}\right)\right]\times\left(150\right)^3$ | | |
| | | $\delta_v=\ -182.057\,mm^3$ | | |
| | | $\boxed{\delta_v=182.057\,mm^3\quad\text{(Decrease)}}$ | 1 | |
| Q. 4 | | **Attempt any two of the following:** | | 16 |
| | (a) | **In a tension test on a certain specimen 25 mm diameter, 250 mm long, an axial pull of 200 kN produces an elongation 0.45 mm and reduction in diameter is observed to be 0.0052 mm. Determine the value of Poisson ratio and three elastic modulii.** | | |
| | Ans. | Given:<br>d = 25 mm<br>L = 250 mm<br>P = 200 kN<br>$\delta_L$ = 0.45 mm<br>$\delta_d$ = 0.0052 mm<br><br>To find:<br>$\mu$ , E, G, K = ? | | |
| | | Solution: | | |
| | | $\text{Poisson's Ratio}=\frac{\text{Lateral Strain}}{\text{Linear Strain}}$ | 1 | |
| | | $=\left(\frac{\left(\frac{\delta_d}{d}\right)}{\left(\frac{\delta_L}{L}\right)}\right)=\left(\frac{\left(\frac{0.0052}{25}\right)}{\left(\frac{0.45}{250}\right)}\right)$ | | |
| | | $\boxed{\text{Poisson's Ratio}\left(\mu\right)=0.115}$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
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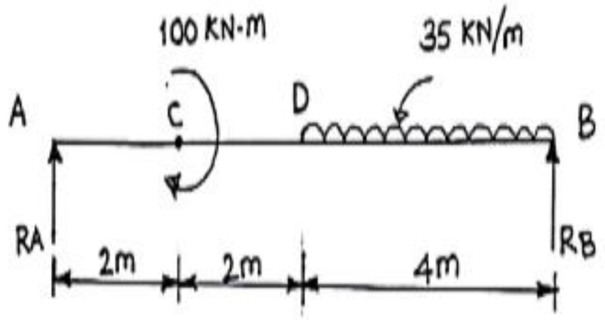
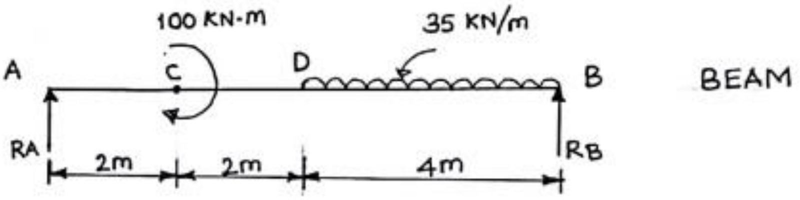
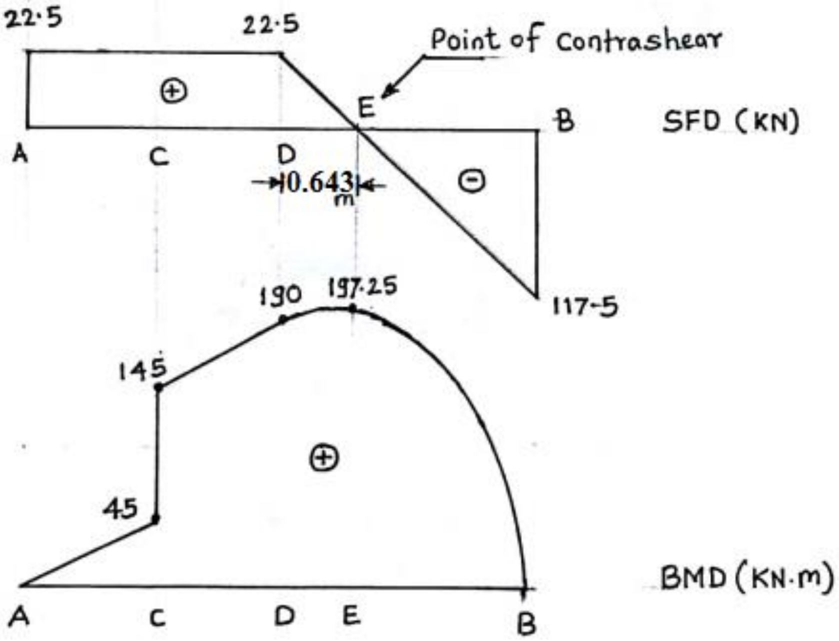
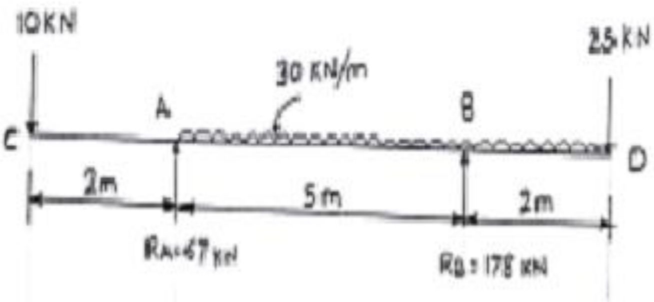
| Q. 4 | | $E=\frac{\sigma}{E}=\left(\frac{\left(\frac{P}{A}\right)}{\left(\frac{\delta_L}{L}\right)}\right)$ | 1 | |
| | | $E=\left(\frac{\left(\frac{200\times10^3}{\frac{\pi}{4}\times25^2}\right)}{\left(\frac{0.45}{250}\right)}\right)$ | | 8 |
| | | $\boxed{E=2.26\times10^5\ N/mm^2}$ | 1 | |
| | | $E=2\times G(1+\mu)$ | 1 | |
| | | $G=\frac{E}{2\times(1+\mu)}=\frac{2.26\times10^5}{2\times(1+0.115)}$ | | |
| | | $\boxed{G=1.015\times10^5\ N/mm^2}$ | 1 | |
| | | $E=3\times K(1-2\times\mu)$ | 1 | |
| | | $K=\frac{E}{3\times(1-2\times\mu)}=\frac{2.26\times10^5}{3\times(1-2\times0.115)}$ | | |
| | | $\boxed{K=0.979\times10^5\ N/mm^2}$ | 1 | |
| | (b) | **A steel rod is subjected to a pull of 15 kN and rigidly fixed at the ends at a certain temperature. Find the magnitude of stress and its nature due to change in temperature by 29° C (both rise and fall). Area of the bar is 250 mm$^2$, E = 210 GPa, α = 12 x 10$^{-6}$/°C.** | | |
| | Ans. | Given:<br>P = 15 kN<br>Δt = 29°C rise<br>Δt = 29°C fall<br>A = 250 mm$^2$<br>E = 210 GPa<br>α = 12 x 10$^{-6}$/°C<br>To find:<br>σ and nature = ?<br><br>Solution: | | |

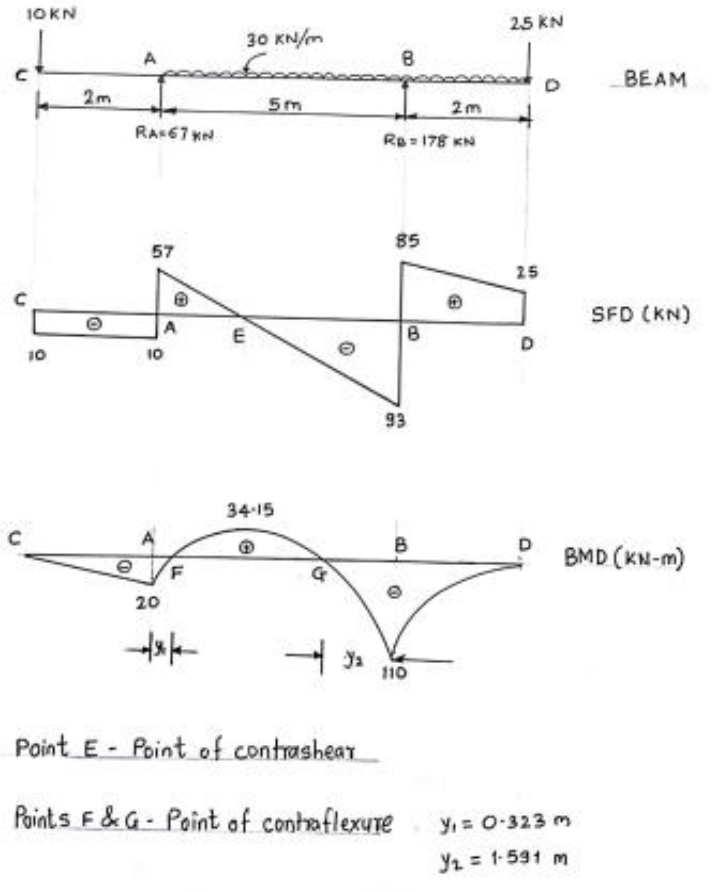
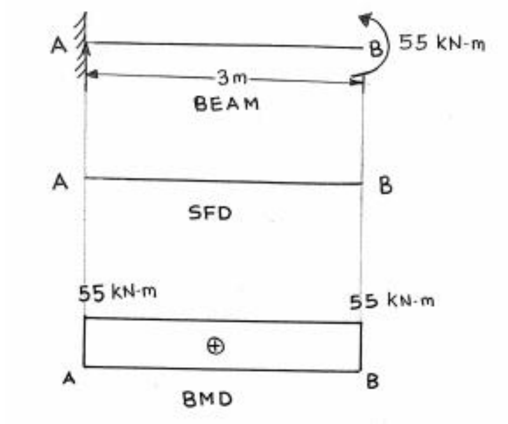
| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 4 | | Case 1 : Rise in temperature | | |
| | | $\sigma_1 = \alpha \times (\Delta t) \times E$ | 1 | |
| | | $= (12\times10^{-6}) \times 29 \times (210\times10^3)$ | | |
| | | $\sigma_1 = 73.08\ N/mm^2\ (C)$ | 1 | |
| | | $\sigma_2 = \frac{P}{A} = \frac{15\times10^3}{250}$ | 1 | |
| | | $\sigma_2 = 60\ N/mm^2\ (T)$ | 1 | 8 |
| | | Net stress $(\sigma_{net}) = \sigma_1 - \sigma_2$ | 1 | |
| | | $= 73.08 - 60$ | | |
| | | $\boxed{\sigma_{net} = 13.08\ N/mm^2\ (C)}$ | 1 | |
| | | Case 2 : Fall in temperature | | |
| | | Net stress $(\sigma_{net}) = \sigma_1 + \sigma_2$ | 1 | |
| | | $= 73.08 + 60$ | | |
| | | $\boxed{\sigma_{net} = 133.08\ N/mm^2\ (T)}$ | 1 | |
| | (c) | **Draw S.F.D. and B.M.D. for a beam loaded as shown in the Fig. No. 01 showing all important values.** | | |
| | |  | | |
| | Ans. | I) To calculate support reactions | | |
| | | $\sum M_A = 0$ | | |
| | | $8\times R_B = 100 + (35\times4)\times6$ | | |
| | | $R_B = 117.50\ kN$ | 1 | |
| | | $\sum F_y = 0$ | | |
| | | $R_A + R_B = (35\times4)$ | | |
| | | $R_A + 117.50 = (35\times4)$ | | |
| | | $R_A = 22.50\ kN$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 4 | | II) SF calculation<br>SF at A =+ 22.50 kN<br>$D_L$ = + 22.50 kN<br>$D_R$ = + 22.50 - 140=+117.50 kN<br>$B_L$ = +117.50 kN<br>B = +117.50 - 117.50=0 kN ( ∴ ok) | 1 | |
| | | III)BM calcualtion<br>BM at A = 0<br>$C_L$ =+(22.5×2) =+45 kNm<br>$C_R$ =+45+100 =+145 kNm<br>D =+(117.50×4) -(35×4×2)=190 kNm<br>B = 0 | 1 | 8 |
| | | IV) To calculate $BM_{max}$<br>SF at E = 0<br>22.5 - (35×X)=0<br>X = 0.643 m from D<br>$BM_{max} = +(117.5\times3.357) - 35\times\frac{3.357^2}{2} = +197.25$ kNm | 1 | |
| | | OR<br>$BM_{max} = +(22.5\times4.643) + 100 - 35\times\frac{0.643^2}{2} = +197.25$ kNm | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 4 | |  | | |
| | |  | 1<br><br>1 | |
| Q. 5 | | **Attempt any TWO of the following:** | | 16 |
| | (a) | **For the beam loaded as shown in the Fig. No. 02 draw S.F.D. and B.M.D. giving all controlling values. Locate the point of cotra-flexure if any.**<br> | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 5 | Ans. | I) To calculate support reactions<br>$\sum M_A = 0$<br>$(10\times2)+(R_B\times5)=\left((30\times7)\times\frac{7}{2}\right)+(25\times7)$<br>$R_B = 178$ kN<br>$\sum F_y = 0$<br>$R_A + R_B = 10+(30\times7)+25$<br>$R_A + 178 = 245$<br>$R_A = 67$ kN | 1 | |
| | | II) SF calculation<br>SF at C $= -10$ kN<br>$C_R = -10$ kN<br>$A_L = -10$ kN<br>$A_R = -10+67 = +57$ kN<br>$B_L = +57-(30\times5) = -93$ kN<br>$B_R = -93+178 = +85$ kN<br>$D_L = +85-(30\times2) = +25$ kN<br>$D = +25 - 25 = 0$ ( $\therefore$ ok) | 1 | 8 |
| | | III) BM calcualtion<br>BM at C $= 0$ ----- (Free end)<br>$A = -(10\times2) = -20$ kNm<br>$B = -(25\times2)-(30\times2)\times1 = -110$ kNm<br>$D = 0$ ----- (Free end) | 1 | |
| | | IV) To calculate $BM_{max}$<br>$B_{max}$ occur at point of contrashear i.e. at E.<br>SF at E = 0<br>$-10 + 67 - 30\times X = 0$<br>$\boxed{X = 1.9 \text{ m from support A}}$ | 1 | |
| | | $B_{max} = -10\times(2+1.9)+(67\times1.9)-(30\times1.9)\times\frac{1.9}{2}$<br>$= +34.15$ kNm | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 5 | | V) To locate point of contraflexure<br>Let F and G are two points of contraflexure.<br>Dist $y_1$ =AF<br>BM at F = 0<br>$-10\times(2+y_1)+(67\times y_1)-\left(30\times\frac{y_1^2}{2}\right)=0$<br>$-20-(10\times y_1)+(67\times y_1)-(15\times y_1^2)=0$<br>$-(15\times y_1^2)+(57\times y_1)-20=0$<br>$(y_1^2)-(3.8\times y_1)+1.33=0$<br>$y_1=\frac{-b\pm\sqrt{b^2-4\times a\times c}}{2\times a}=\frac{3.8\pm\sqrt{-(3.8)^2+(4\times1\times1.33)}}{2\times1}$<br>Solving the equation<br>$y_1$=0.39 m from support A | ½ | |
| | | BM at G = 0<br>$-25\times(2+y_2)+(178\times y_2)-\left(30\times\frac{(2+y_2)^2}{2}\right)=0$<br>$-50-(25\times y_2)+(178\times y_2)-(15\times(2+y_2)^2)=0$<br>$-(15\times(4+4y_2+y_2^2))+(153\times y_2)-50=0$<br>$-(15\times y_2^2)+(93\times y_2)-110=0$<br>$(y_2^2)-(6.2\times y_2)+7.33=0$<br>$y_2=\frac{-b\pm\sqrt{b^2-4\times a\times c}}{2\times a}=\frac{6.2\pm\sqrt{(-6.2)^2-(4\times1\times7.33)}}{2\times1}$<br>Solving the equation<br>$y_2$=1.591m from support B | ½ | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 5 | |  | 1 (SFD)<br>1 (BMD) | |
| | (b)(i) | **A cantilever beam of 3 m span is carrying anticlockwise moment of 55 kNm at its free end. Draw S.F.D. and BM.D. showing all values.** | | |
| | Ans. | SF : Since there is no load on the beam SF at any section between A to B is zero. Therefore SF is horizontal straight line.<br>BM : Bending moment at section x-x at a distance x from B is –<br>$M_x$ = + 55 kNm<br>Hence, $M_A = M_B$ = + 55 kNm | 1<br><br>1 | |
| | |  | 1 (SFD)<br>1 (BMD) | 4 |

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>Q. 5</td><td>(b)(ii)</td><td>

**A cantilever beam of span 3.5 m has u.d.l. of 20 kN/m throughout the span along with a downward point load of 20 kN at its free end. Draw S.F.D. and BM.D. showing all values.**

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<tr><td></td><td>Ans.</td><td>

I) Reaction at A

$R_A = 20+(20\times3.5)=90\,kN$

II) SF calculations

SF at A = + 90 kN

$B_L = +90-(20\times3.5) = +20\,kN$

$B_R = +20-20 = 0 \;(\therefore ok)$

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III) BM calculations

BM at B = 0 ------(Free end)

$A = -(20\times3.5) - (20\times3.5)\times\frac{3.5}{2} = -192.50\,kNm$

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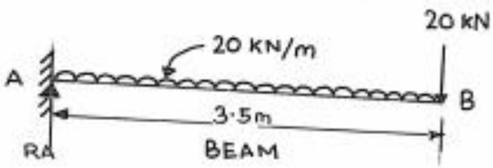


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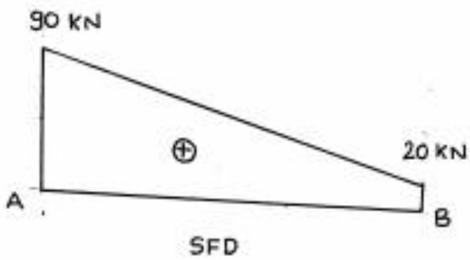


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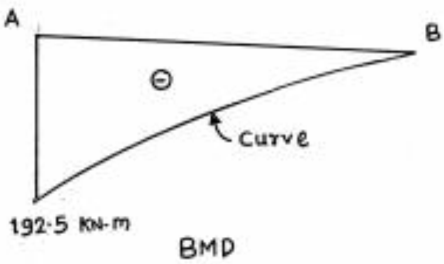


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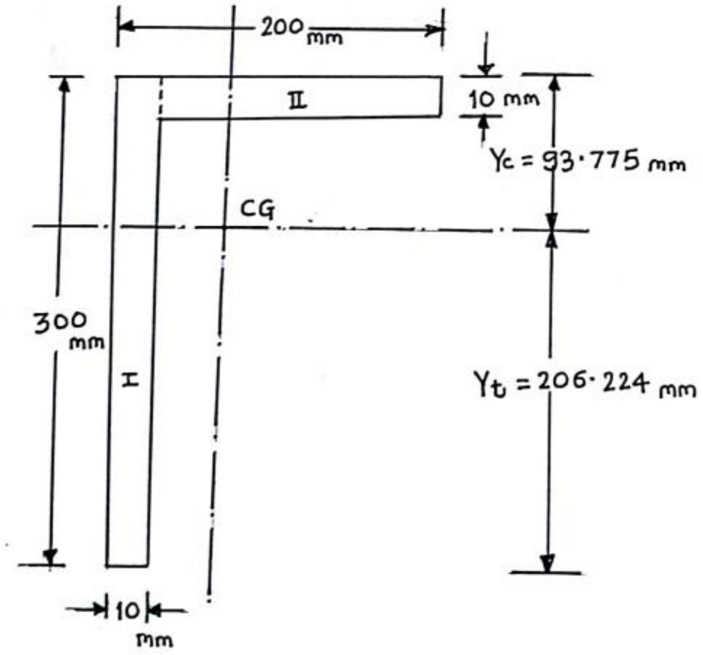
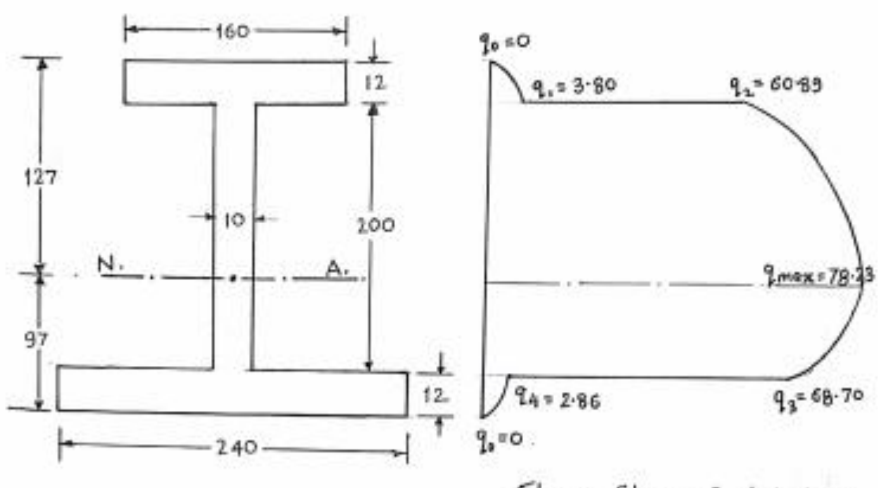
| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 5 | (c) | **The cross section of a simply supported beam is as shown in the figure No. 03. The permissible bending stresses in tension and compression are 200 MPa and 140 MPa respectively. Determine the moment of resistance.** | | |
| | Ans. |   | | |
| | | I) To calculate $\bar{y}$ | | |
| | | $\bar{y}=\frac{(a_1\times y_1)+(a_2\times y_2)}{(a_1+a_2)}$ | | |
| | | $=\frac{\left[(10\times300)\times\frac{300}{2}\right]+\left[(190\times10)\times\left(300-\frac{10}{2}\right)\right]}{[(10\times300)+(190\times10)]}$ | | |
| | | $=206.224\text{ mm from base}$ | 1 | |
| | | $y_t=206.224\text{ mm}$ | ½ | |
| | | $y_c=300-206.224=93.775\text{ mm}$ | | |
| | | II) To calculate $I_{xx}$ | | |
| | | $h_I=206.224-\frac{300}{2}=56.224\text{ mm}$ | 1 | |
| | | $h_{II}=93.775-\frac{10}{2}=88.775\text{ mm}$ | | |
| | | $I_{xx}=[I_{xx}]_I+[I_{xx}]_{II}$ | | 8 |
| | | $=\left[\frac{b\times d^3}{12}+(a\times h^2)\right]_I+\left[\frac{b\times d^3}{12}+(a\times h^2)\right]_{II}$ | 1 | |
| | | $=\left[\frac{10\times300^3}{12}+(3000\times56.224^2)\right]_I+\left[\frac{190\times10^3}{12}+(1900\times88.775^2)\right]_{II}$ | | |
| | | $I_{xx}=46973149.05\text{ mm}^4=46.973\times10^6\text{ mm}^4$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 5 | | III) To calculate Moment of resistance | | |
| | | $M_R = \left(\frac{\sigma_t}{y_t}\right) \times I_{xx}$ | ½ | |
| | | $= \left(\frac{200}{206.224}\right) \times 46973149.05$ | | |
| | | $M_R = 45555463.04\ Nmm = 45.555 \times 10^6\ Nmm$ | 1 | |
| | | $M_R = \left(\frac{\sigma_c}{y_c}\right) \times I_{xx}$ | ½ | |
| | | $= \left(\frac{140}{93.775}\right) \times 46973149.05$ | | |
| | | $M_R = 70127868.48\ Nmm = 70.127 \times 10^6\ Nmm$ | 1 | |
| | | Take minimum value of $M_R$ from above | | |
| | | $\boxed{M_R = 45555463.04\ Nmm = 45.555 \times 10^6\ Nmm}$ | ½ | |
| Q. 6 | | **Attempt any TWO of the following:** | | 16 |
| | (a) | **A beam of unsymmetrical I – section has the following details: Top flange – 160 mm x 12 mm, bottom flange – 240 mm x 12 mm, web – 10 mm x 200 mm, overall depth = 224 mm. The centroid of the section is at a distance of 97 mm from the base and $I_{xx} = 59.13 \times 10^6\ mm^4$. Draw the shear stress distribution diagram at a section where shear force is 155 kN.** | | |
| | Ans. |  Shear Stress Distribution | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 6 | | I) At top extreme fibre $(q_0)=0$ | ½ | |
| | | II) At bottom of top flange $(q_1)$ <br> $=\frac{S\times A\times \overline{Y}}{b\times I}$ <br> $=\frac{(155\times10^3)\times(160\times12)\times121}{160\times(59.13\times10^6)}$ <br> $=3.806\ N/mm^2$ | 1 | |
| | | III) At junction of top flange and web $(q_2)$ <br> $=\frac{S\times A\times \overline{Y}}{b\times I}$ <br> $=\frac{(155\times10^3)\times(160\times12)\times121}{10\times(59.13\times10^6)}$ <br> $=60.899\ N/mm^2$ | 1 | |
| | | IV) At N.A. $(q_{max})$ <br> $=\frac{S\times(A\times \overline{Y})}{b\times I}$ <br> $a_1=(160\times12)=1920\ mm^2$ <br> $a_2=(115\times10)=1150\ mm^2$ <br> $y_1=127-6=121\ mm$ <br> $y_2=\left(\frac{(127-12)}{2}\right)=57.50\ mm$ | 1 | 8 |
| | | $(A\times\overline{Y})=(a_1\times y_1)+(a_2\times y_2)$ <br> $=\left[(160\times12)\times121\right]+\left[(115\times10)\times57.5\right]$ <br> $=298445$ <br> $=\frac{S\times(A\times \overline{Y})}{b\times I}$ <br> $=\frac{(155\times10^3)\times(298445)}{10\times(59.13\times10^6)}$ <br> $=78.232\ N/mm^2$ | 1 | |
| | | V) At junction of bottom flange and web $(q_3)$ <br> $=\frac{S\times A\times \overline{Y}}{b\times I}$ <br> $=\frac{(155\times10^3)\times(240\times12)\times91}{10\times(59.13\times10^6)}$ <br> $=68.70\ N/mm^2$ | 1 | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 6 | | VI) At junction of bottom flange ($q_4$) <br> $=\frac{S\times A\times \overline{Y}}{b\times I}$ <br> $=\frac{(155\times10^3)\times(240\times12)\times91}{240\times(59.13\times10^6)}$ <br> $=2.862\ N/mm^2$ | 1 | |
| | | VII) At bottom extreme fibre ($q_0$)=0 | ½ | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 6 | (b) | **A certain column of hollow circular section has an external diameter 300 mm and metal thickness 40 mm. The column is 6 m long having one end fixed and one end hinged. Find the safe load for the column using Rankine's formula. Use a factor of safety of 8. Take $\sigma_c$ = 567 MPa and $\alpha$ = 1/1600.** | | |
| | Ans. | Given: D = 300 mm, t = 40 mm, L = 6 m = 6000 mm, $\sigma_c$ = 567 MPa, $\alpha$ = 1/1600, FOS = 8. To find: $P_{safe}$ = ? | | |
| | | Solution: | | |
| | | I) $L_e=\frac{L}{\sqrt{2}}=\frac{6000}{\sqrt{2}}=4242.64\,mm$ | 1 | |
| | | II) $d=(D-2\times t)=(300-2\times 40)=220\,mm$ | ½ | |
| | | III) $A=\frac{\pi}{4}\times(D^2-d^2)=\frac{\pi}{4}\times(300^2-220^2)=32672.5636\,mm^2$ | ½ | |
| | | IV) $I_{min}=\frac{\pi}{64}\times(D^4-d^4)=\frac{\pi}{64}\times(300^4-220^4)=282617675.1\,mm^4$ | 1 | |
| | | V) $K_{min}=\sqrt{\frac{I_{min}}{A}}=\sqrt{\frac{282617675.1}{32672.5636}}=93\,mm$ | 1 | |
| | | OR | | |
| | | $K=\sqrt{\frac{D^2+d^2}{16}}=\sqrt{\frac{300^2+220^2}{16}}=93\,mm$ | | 8 |
| | | VI) $\lambda^2=\left[\frac{L_e}{K_{min}}\right]^2=\left[\frac{4242.64}{93}\right]^2=2081.165\,mm$ | ½ | |
| | | VII) By Rankine's formula | | |
| | | $P_R=\frac{\sigma_c\times A}{1+(\alpha\times\lambda^2)}$ | 1 | |
| | | $=\frac{567\times 32672.5636}{1+\left(\frac{1}{1600}\times 2081.165\right)}$ | | |
| | | $=8052472.57\,N$ | 1 | |
| | | VIII) $P_{safe}=\frac{P_R}{FOS}$ | 1 | |
| | | $=\frac{8052472.57}{8}$ | | |
| | | $\boxed{P_{safe}=1006559.071\,N}$ | ½ | |
| | | $\boxed{P_{safe}=1006.559\,kN}$ | | |

| Que. No. | Sub. Que. | Model Answers | Marks | Total Marks |
|---|---|---|---|---|
| Q. 6 | (c) | **A bar 22 mm in diameter and 1.2 m long is hung vertically and a collar is attached at the lower end. A weight of 1000 N falls through a height of 275 mm on the collar. Calculate maximum instantaneous stress, elongation and the strain energy stored in the bar. Take E = 210 GPa.** | | |
| | Ans. | Given: d = 22 mm, L = 1.2 m = 1200 mm, W = 1000 N, h = 275 mm, E = 210 GPa. To find: $\sigma_{max}$, $\delta_L$, U = ? | | |
| | | Solution: | | |
| | | I) $A=\frac{\pi}{4}\times d^2=\frac{\pi}{4}\times 22^2=380.133\,mm^2$ | ½ | |
| | | $V=A\times L=380.133\times 1200=456159.2533\,mm^3$ | ½ | |
| | | II) $\sigma_{max}=\frac{W}{A}+\sqrt{\left(\frac{W}{A}\right)^2+\left(\frac{2\times W\times h\times E}{A\times L}\right)}$ | 1 | |
| | | $=\frac{1000}{380.133}+\sqrt{\left(\frac{1000}{380.133}\right)^2+\left(\frac{2\times 1000\times 275\times 210\times 10^3}{456159.2533}\right)}$ | 1 | |
| | | $\boxed{\sigma_{max}=505.828\,N/mm^2}$ | 1 | 8 |
| | | III) $\delta_L=\frac{\sigma_{max}\times L}{E}$ | 1 | |
| | | $=\frac{505.828\times 1200}{210\times 10^3}$ | | |
| | | $\boxed{\delta_L=2.89\,mm}$ | 1 | |
| | | IV) $U=\frac{\sigma_{max}^2}{2\times E}\times V$ | 1 | |
| | | $=\frac{505.828^2}{2\times 210\times 10^3}\times 456159.2533$ | | |
| | | $U=277890.0075\,Nmm$<br>$U=277.89\,Nm$ or J | 1 | |